Proof by Induction

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How do I prove an infinite family of statements? Luckily, mathematicians have devised a way to do so. It is called "induction." It relies on the premise that such statements are countably infinite. In other words, we may attach a number to every such statements. It is common to name the statement after the number we associate with it. For example, we may say the 1st statement is $P(1)$
So if we want to actually wish to implement induction, we require to show that $P(1)$ holds and that whenever $P(k)$ holds, we have that $P(k+1)$ also holds. Here's an example of induction in action.

Prove that $P(n) =$ "$1 + 2 + 3 + \ldots n = \frac{n(n+1)}{2}$."

Proof (by induction):
Base case: Consider $n = 1$, we then have:
$1 = \frac{1*(1+1)}{2}$
$1 = 1$
We then proceed with the inductive step.
Assume that $P(k)$ holds, that is:
$1 + 2 + 3 + \ldots k = \frac{k(k+1)}{2}$
We wish to show that from $P(k)$, $P(k+1)$ holds.
We know add both sides by $k + 1$ to obtain:
$1 + 2 + 3 + \ldots k + (k+1) = \frac{k(k+1)}{2} + (k+1)$
$1 + 2 + 3 + \ldots k + (k+1) = \frac{k(k+1)}{2} + \frac{2k+2}{2}$
$1 + 2 + 3 + \ldots k + (k+1) = \frac{k^2+3k+2}{2}$
$1 + 2 + 3 + \ldots k + (k+1) = \frac{(k+1)(k+2)}{2}$
We have verified that $P(k+1)$ holds.