Lie Algebra

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Hello everybody! This week I will do a quick introduction to Lie Algebras, which is the study of vector spaces with an operation called the Lie Bracket.(Lie is pronounced Lee) It's motivated by the study of Lie Groups in Differential Geometry and also plays an important role in Representation Theory.

Definition of a Lie Algebra: Let V be a non-empty vector space over some field F. I denote an operation $[\times, \times]$ as a map from $V \times V \to V$ which I call, the Lie Bracket as long as it satisfies the following conditions.
1. It is bilinear. That is, if $x, y, z \in V$ and $a, b \in F$, then
$[ax + by, z] = a[x, z] + b[y, z]$
$[z, ax + by] = a[z, x] + b[z, y]$
2. We require that $\forall x \in V$, we have $[x, x] = 0$.
3. It is also necessary that bracket satisfy the Jacobi identity, which is
$[x, [y, z]] + [y, [z, x]] + [z, [y, x]] = 0$